Simple Stresses & Strains

Introduction :- When an external force acts on a body, the body tends to undergo some deformation. Due to cohesion between the molecules, the body resists deformation. This resistance by which material of the body opposes the deformation is known as Strength Of Material.

Stress :- The force of resistance per unit area, offered by a body against deformation is known as stress. The external force acting on the body is called the load. A loaded member remains in equillibrium when the resistance offered by the member against the deformation & the applied load are equal.

$$\sigma = \frac{P}{A}$$

Where σ = Stress (also called intensity of stress)

A = Area of cross-section

P = External force or load

Unit of stress is N/m^2 or N/cm^2 or N/mm^2

1 Pascal = $1 N/m^2$

Strain :- When a body is subjected to some external force, there is some change of dimension of body. The ratio of change of dimension of body to the original dimension is known as strain.

Strain may be

1. Tensile strain
2. Compressive strain
3. Shear strain
4. Volumetric strain

TYPES OF STRESSES :-

1. TENSILE STRESS
2. COMPRESSIVE STRESS
3. SHEAR STRESS

1. TENSILE STRESS :- The stress induced in a body, when subjected to two equal & opposite pulls as a result of which there is an increase in length, is known as tensile stress. The ratio of increase in length to the original length is called tensile strain.

Let P = Pull (or force) acting on the body

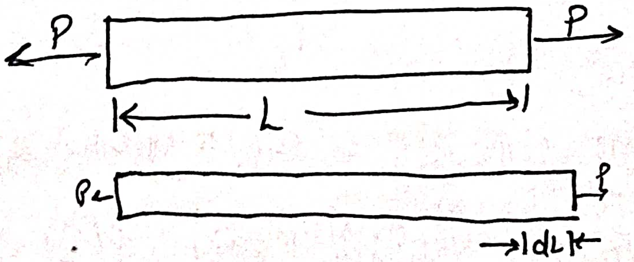
A = Cross-sectional area of the body

L = Original length of the body

dL = Increase in length due to pull P acting on the body

σ = Stress induced in the body, &

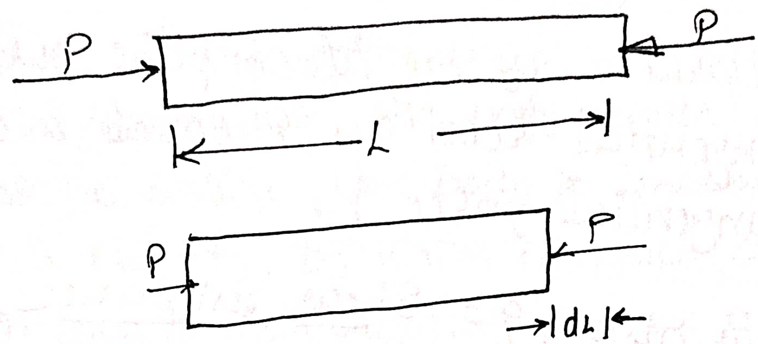
e = Strain (i.e. tensile strain)



Tensile stress, $\sigma_t = \frac{P}{A}$

Tensile strain, $e_t = \frac{dL}{L}$

COMPRESSIVE STRESS :- The stress induced in a body, when subjected to two equal & opposite pushes as a result of which there is decrease in length of the body, is known as compressive stress. And also the ratio of decrease in length to the original length is known as compressive strain.



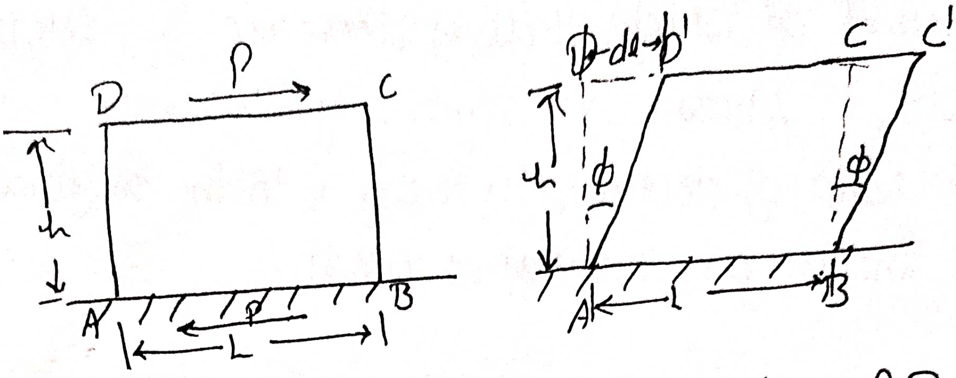
Let an axial push P is acting on a body of cross-sectional area A. Due to external push P, let the original length L of the body decreases by dL.

$\therefore$ Compressive stress $\sigma_c = \dfrac{P}{A}$

& Compressive strain $e_c = \dfrac{\text{Decrease in Length}}{\text{Original Length}} = \dfrac{dL}{L}$

3. **SHEAR STRESS** :- The stress induced in a body, when subjected to two equal & opposite forces which are acting tangentially across the resisting section as a result of which the body tends to shear off across the section is known as shear stress. The corresponding strain is known as shear strain.

Consider a rectangular block of height h, length L, & width unity. Let the bottom face AB of the block be fixed to the surface. Let a force P be applied tangentially along the top face CD of the block.



For the equilibrium of the block, the surface AB will offer a tangential reaction P equal & opposite to applied tangential force P.

$\therefore$ Shear stress, $q = \frac{\text{Shear resistance}}{\text{Area}} = \frac{P}{A}$

$= \frac{P}{L \times 1}$

As the bottom face of the block is fixed, the face ABCD will distorted to ABC_1D_1 through an angle ϕ as a result force P. &

Shear strain(ϕ) is given by

$\phi = \frac{\text{Transverse Displacement}}{\text{Distance AD}}$

$= \frac{DD_1}{AD}$

$\boxed{\phi = \frac{dl}{h}}$

HOOK'S LAW :- It states that when a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress. This means the ratio of the stress to corresponding strain is a constant with in elastic limit. This constant is known as Modulus of Elasticity or Modulus of Rigidity.

ELASTIC LIMIT :- When an external force acts on a body, the body tends to undergo some deformation. If the external force is removed & the body comes back to its original shape & size, the body is known as elastic body. This property, by virtue of which certain materials return back to their original position after the removal of the external force is called elasticity.

There is a limiting value of force up to & within which, the deformation completely disappears on the removal of the force. The value of stress corresponding to this limiting force is known as the elastic limit of the material.

ELASTIC CONSTANTS :-

1. Modulus of elasticity :- The ratio of tensile stress or compressive stress to the corresponding strain is a constant. This ratio is known as Young's Modulus or Modulus of Elasticity & is denoted by E.

$$\therefore E = \frac{\text{Tensile stress}}{\text{Tensile strain}} \text{ or } \frac{\text{Compressive stress}}{\text{Compressive strain}}$$

$$E = \frac{\sigma}{e}$$

2. Modulus of rigidity or Shear Modulus :- The ratio of shear stress to the corresponding shear strain within elastic limit, is known as modulus of rigidity.

$$C \text{ or } G = \frac{q}{\phi} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

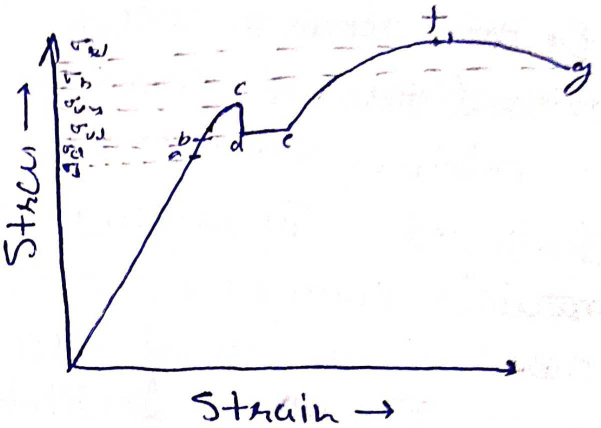
3. <u>Bulk Modulus</u>:- (K) It is defined as ratio of direct stress to volumetric strain.

$$K = \frac{\sigma}{\frac{dV}{V}} = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

<u>STRESS - STRAIN DIAGRAM</u> :-

(a) for <u>DUCTILE MATERIAL</u>:-

<u>Point a</u>:- This point is called proportional limit point. The stress corresponding to this point is called proportional limit stress. Up to proportional limit point stress is directly proportional to strain.



<u>Point b</u> :- This point is called elastic limit point. The stress corresponding to this point is called elastic limit (σ_{el}) stress. There is a limiting value of force up to & within which, the deformation completely disappears on the removal of force. The value of stress corresponding to this limiting force is known as elastic limit of the material.

<u>Point C</u> :- This point is called upper yield point. The stress corresponding to this point is called upper yield stress (σ_{uy}). After point b, plastic deformation starts.

<u>Point d</u> :- This point is called lower yield point.

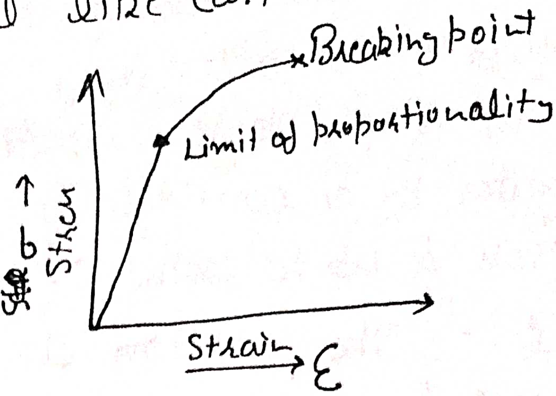
The stress corresponding to this point is known as lower yield stress (σ_{yl}). From point c to d, the value of stress decreases. At point d the specimen elongates by a considerable amount without any increase in stress & up to point e.

Point e :- The portion de is called the yielding of material at constant stress. From point e onwards, the strain hardening phenomenon becomes predominant & the strength of material increases thereby requiring more stress for deformation, until point f is reached.

Point f :- Point f is called ultimate point & the stress corresponding to this point is called ultimate stress (σ_u). It is maximum stress to which the material can be subjected in simple tensile test. At point f, the necking of material begins & the cross-sectional area starts decreasing at a rapid rate. Due to this local necking, the stress in the material goes on decreasing inspite of fact that actual stress intensity goes on increasing.

Point g :- Ultimately the specimen breaks at point g, known as breaking point, & the corresponding stress is called the nominal breaking stress based upon the original area of cross-section.

(b) for brittle materials :- The stress-strain diagram for a brittle material like cast iron is as shown. There is very little elongation & reduction in area of the specimen for such materials. The yield point is not marked at all.



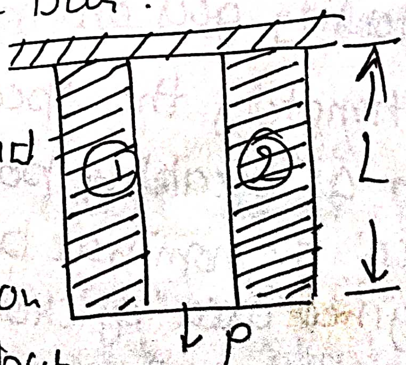
* Composite System :-

A system consisting of more than one bar or tube of the same or different material rigidly connected in such a way that when subjected to loads or variation in temperature, each individual component undergoes equal changes in length is called a composite system.

ANALYSIS OF BARS OF COMPOSITE SECTIONS :-

A bar, made up of two or more bars of equal lengths but of different materials rigidly fixed with each other & behaving as one unit for extension or compression when subjected to an axial tensile or compressive load is a composite bar.

For the composite bar the following two points are important



1. The extension or compression in each bar is equal. Hence deformation per unit length i.e. strain in each bar is equal. $\varepsilon_1 = \varepsilon_2$

2. The total external load on the composite bar is equal to the sum of loads carried by each different material $\boxed{P = P_1 + P_2}$

where P = Total load on composite bar

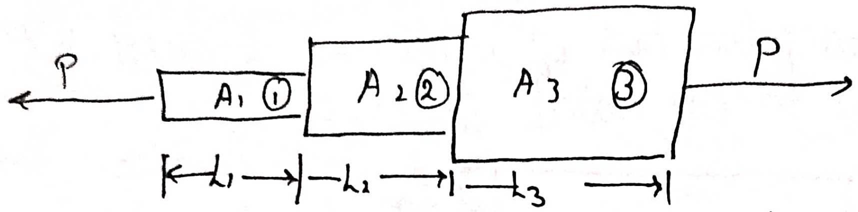
L = length of composite bar

P_1 = Load shared by bar 1

P_2 = Load shared by bar 2

ANALYSIS OF BARS OF VARYING SECTIONS

A bar of different lengths & of different diameters (& hence of diff. cross-sectional areas) is subjected to an axial load P.



Let P = Axial Load acting on the bar

L_1 = Length of section 1,

A_1 = Cross-sectional area of section 1,

L_2, A_2 = Length & cross-sectional area of section 2.

L_3, A_3 = Length & cross-sectional area of section 3, &

E = Young's Modulus of bar

Then stress for section 1

$$\sigma_1 = \frac{P}{A_1}$$

Similarly for section 2 & 3

$$\sigma_2 = \frac{P}{A_2}, \quad \sigma_3 = P/A_3$$

& Strain for section 1, $e_1 = \frac{P}{A_1 E}$

Section 2, $e_2 = \frac{P}{A_2 E}$

$$e_3 = \frac{P}{A_3 E}$$

Change in length of section 1, $dL_1 = e_1 L_1$

$$= \frac{P L_1}{A_1 E}$$

Similarly $dL_2 = \frac{P L_2}{A_2 E}$

& $dL_3 = \frac{P L_3}{A_3 E}$

Total change in the length of the bar

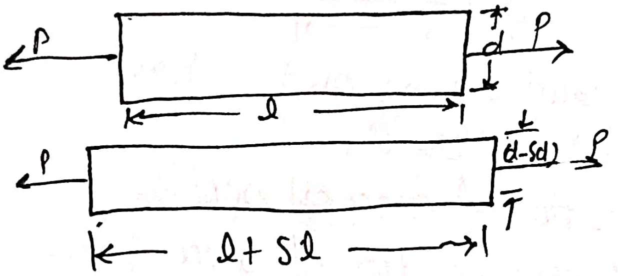
$$dL = dL_1 + dL_2 + dL_3$$

$$= \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E}$$

$$\boxed{dL = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)}$$

LATERAL STRAIN:- The strain at right angles to the direction of applied load is known as lateral strain.

Let a rectangular bar of length L, breadth b & depth d is subjected to an axial load P



The length of bar will increase while the breadth & depth will decrease.

Let δL = Increase in length

δb = Decrease in breadth, &

δd = Decrease in depth

Then, Longitudinal strain $= \frac{\delta L}{L}$

& Lateral strain $= \frac{\delta b}{b}$ or $\frac{\delta d}{d}$

Poisson's ratio:- The ratio of lateral strain to the longitudinal strain is a constant for a given material when the material is stressed within elastic limit. This ratio is called Poisson's ratio.

$$\boxed{\text{Poisson's ratio } (\mu \text{ or } \tfrac{1}{m}) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}}$$

...ic Constants & Thier Relationship

E , G & K

E – Young's Modulus of Elasticity = $\frac{\text{Longitudinal stress}}{\text{" strain}}$

G – Modulus of rigidity or shear Modulus, $\frac{\text{Shear stress}}{\text{" strain}}$

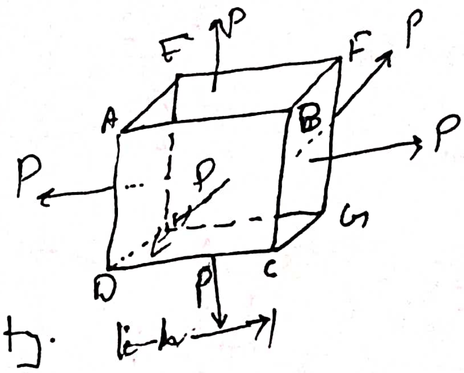
K = Bulk Modulus = $\frac{\text{Direct stress}}{\text{Volumetric strain}}$

$$\boxed{E = 2G\left(1+\frac{1}{m}\right) = 3K\left(1-\frac{2}{m}\right)}$$

$\frac{1}{m}$ → Poison ratio

Relationship b/w E & K :-

Consider a cube ABCDEFGH which is subjected to three mutually perpendicular tensile stresses of equal intensity.



L = length of cube

dL = Change in length of the cube

E = Young's modulus of the material of the cube

$\frac{1}{m}$ = Poisson's ratio

Volume of cube, $V = L^3$

Now let us consider the strain of one of sides of the cube (say AB) under the action of three mutually perpendicular stresses.

1. Strain of AB due to elongation $e_1 = \frac{\sigma}{E}$
2. Strain of AB due to compression of BF, $e_2 = -\frac{1}{m}\frac{\sigma}{E}$
3. Strain of AB due to compression of BC, $e_3 = -\frac{1}{m}\frac{\sigma}{E}$

Total strain of AB is given by

$$\frac{dL}{L} = \frac{\sigma}{E} - \frac{1}{m}\frac{\sigma}{E} - \frac{1}{m}\frac{\sigma}{E}$$

$$\frac{dL}{L} = \frac{\sigma}{E}\left(1-\frac{2}{m}\right) \quad \text{--- (1)}$$

$$V = L^3$$

Differentiating it

$$dV = 3L^2 \, dL$$

$$\frac{dV}{V} = \frac{3L^2 \, dL}{V}$$

$$\Rightarrow \frac{dV}{V} = \frac{3L^2 \, dL}{L^3}$$

$$\Rightarrow \frac{dV}{V} = \frac{3 \, dL}{L} \quad \text{--- (2)}$$

From eqn (1) & (2)

$$\frac{dV}{V} = 3 \frac{\sigma}{E}\left(1 - \frac{2}{m}\right)$$

$$E = 3 \frac{\sigma}{\frac{dV}{V}}\left(1 - \frac{2}{m}\right)$$

$$\boxed{E = 3K\left(1 - \frac{2}{m}\right)}$$

Comboand n theA

Column X

BMSFX

Thz